MATH 122
MATHEMATICAL INDUCTION PROBLEMS

Use induction to prove that each of the following formulas is true for each positive integer n.

1. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$
2. $1 \bullet 2 + 3 \bullet 4 + 5 \bullet 6 + \ldots + (2n-1)(2n) = \dfrac{n(n+1)(4n-1)}{3}$
3. $\dfrac{1}{2} + \dfrac{2}{2} + \dfrac{3}{2} + \ldots + \dfrac{n}{2} = \dfrac{n(n+1)}{4}$
4. $2 + 6 + 10 + \ldots + 4n - 2 = 2n^2$
5. $2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$
6. $1 \bullet 2 + 2 \bullet 3 + 3 \bullet 4 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$
7. $1 \bullet 2^2 + 2 \bullet 3^2 + 3 \bullet 4^2 + \ldots + n(n+1)^2 = \dfrac{1}{12} n(n+1)(n+2)(3n+5)$

By induction show that:

8. $3^n - 1$ is divisible by 2.
9. $5^n - 1$ is divisible by 4.
10. $7^n - 1$ is divisible by 6.
11. $8^{2n} - 1$ is divisible by 63.
12. $6^{2n} - 1$ is divisible by 35.
13. $9^{2n} - 1$ is divisible by 80.
14. $n^2 - 3n + 4$ is even.
15. $2n^3 - 3n^2 + n$ is divisible by 6.
16. a. Show: If $2 + 4 + 6 + \ldots + 2n = n(n+1) + 2$ is true for $n = j$, then it is true for $n = j + 1$.

    b. Is the formula true for all n?